GENERAL TOPOLOGY

Summer Bailey

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Topology

Your sitting in a coffee shop looking at your coffee cup with a handle and your donut. A regular person may think these two things are completely different but to a topologist these two objects are equivalent. Since these two objects can be continuously deformed into one another, they are topologically equal. This is the concept of topology, in which two objects are equivalent when they can be continuously deformed into one another through bending, twisting, stretching, and shrinking. Topology is sometimes referred to as "rubber sheet geometry" because of the properties that it possesses. Topology applies to physics and our world around, but then again explains things that do not exist in our world. Topologists such as Klein and Mobius, have made some important discoveries in topology.

First to understand topology, an understanding of its definitions in a must. These definitions include topology, open set, closed set, homeomorphism, continuous function, compact space, and connected space. The formal definition of a topology is as follows:

A topology on a set X is a collection i of subsets of X, called the open sets, satisfying:
1. Any union of elements of i belongs to i
2. Any finite intersection of elements of i belongs to i
3. none- empty set and X belong to i

We say (X,i) is a topological space, sometimes abbreviated X is a topological space. All topologies have to follow these 3 rules. In topology, the sets are either defined as open or closed. An open set is a set that has a neighborhood for every point in the set. In two-space the open set is a disk. A closed set is just the opposite of an open set since it is defined as it contains all of its limit points. This means that a closed set has a boundary.

In topology to be topologically equivalent, there exists a function known as a homeomorphism, h. To have two objects , X and Y, be homeomorphic to one another, the function has to fulfill these conditions:
1. h is a one- to one function between X and Y
2. h is continuous
3. there exists a continuous inverse function

In general topology or otherwise known as point-set topology has three fundamental concepts that are continuous functions, compact space, and connected space.

A continuous function is defined as:
A function f mapping X to Y is continuous if for each open set V of Y, the subset of X consisting of all points p for which f(p) belongs to V is an open set of X. This means that

for every p in X, there is a point in V that corresponds to an open set in X.

A compact space in a topology is:
Let X be a topological space. A collection Gi of open subsets of X is said to be an open cover of X if each point in X belongs to at least one Gi. A subclass of an open cover which is itself an open cover is called a sub cover. A compact space is a topological space in which every open cover has a finite subcover.

A connected space in topology is:
A connected space is a topological space X which cannot be represented as the union of two disjoint non-empty open sets. If X=A union B, where A and B are disjoint and open, then A and B are also closed, so that X is the union of two disjoint closed sets. We see by this that X is connected double arrow it cannot be represented as the union of two disjoint non-empty closed sets.

The definitions of topology, open set, closed set, homeomorphism, continuous function, compact space and connected space are the fundamental definitions of general topology. To understand any high topology such as differential topology, an understanding of these is required.

The mathematician that laid the groundwork for topology was, to no surprise Gottfried Wilhelm Leibniz. In 1679 he proposed his idea for analysis situs, now known as general topology. Although Leibniz founded general topology, after that he didnt have any results to display for topology. Instead, two prominent mathematicians in the field of topology is Augustus Ferdinand Mobius and Felix Klein. Both of these mathematicians are known for one important result in the area of topology. Mobius invented the Mobius strip and Klein developed the paradoxical figure, the Klein bottle. Augustus Mobius was a mathematician, astronomer, developer, publisher, and inventor. He invented the four- color conjecture, Mobius transformations, Mobius tetrahedrons, Mobius net, Mobius function and the Mobius strip just to name a few. He is now remembered for his discovery of the Mobius strip, which is formed by taking a rectangular strip of paper and connecting the ends after giving it a half twist. This results in a one- sided, none orientable object because when drawing a straight, continuous line on the Mobius strip, the line will cover the entire length of the strip without even crossing an edge, and it will return to its starting point in one long stroke. This is an important contribution to topology because it is a surface with boundary and has a continuous function describing the path of the line on the surface. Therefore it is a closed topological space with a continuous function. The Mobius strip is used in the real world as a conveyer belt that lasts twice as long as a conventional belt. A Mobius strip can also be illustrated as interlocking turning gears along the length of the strip.

In turn, another prominent mathematician for topology is Felix Klein. He largely contributed to the area of topology by his discovery of the Klein bottle. Just like the Mobius strip, the Klein bottle is a two dimensional, none orientable object but it is different because it is an compact space without boundary, an open set. The Klein bottle can be embedded in R4, which makes it so hard to understand. A description of the Klein bottle is: It is a tapered tube whose neck is bent around to enter the side of the bottle. The neck continuous into the base of the bottle where it flares out and rounds off to form the outer surface of the bottle. Just like the Mobius strip, it has a continuous function that describes the path of the line on the surface of the Klein bottle. So the Klein bottle is a open, compact topological space with a continuous function. The application of the Klein bottle is out of our reach since it is in R4 without intersecting itself, but in R3 we can make the Klein bottle with it intersecting itself. This is a major contribution to topology since it is an example of a compact, open topological space.